The Concept Outline

Big Idea 1: Limits

Many calculus concepts are developed by first considering a discrete model and then the consequences of a limiting case. Therefore, the idea of limits is essential for discovering and developing important ideas, definitions, formulas, and theorems in calculus. Students must have a solid, intuitive understanding of limits and be able to compute various limits, including one-sided limits, limits at infinity, the limit of a sequence, and infinite limits. They should be able to work with tables and graphs in order to estimate the limit of a function at a point. Students should know the algebraic properties of limits and techniques for finding limits of indeterminate forms, and they should be able to apply limits to understand the behavior of a function near a point. Students must also understand how limits are used to determine continuity, a fundamental property of functions.

CURRICULUM FRAMEWORK

Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 1.1: The concept of a limit can be used to understand the behavior of functions.	**LO 1.1A(a):** Express limits symbolically using correct notation. **LO 1.1A(b):** Interpret limits expressed symbolically.	**EK 1.1A1:** Given a function f, the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x\to c} f(x) = R$. **EXCLUSION STATEMENT (EK 1.1A1):** *The epsilon-delta definition of a limit is not assessed on the AP Calculus AB or BC Exam. However, teachers may include this topic in the course if time permits.*
		EK 1.1A2: The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.
		EK 1.1A3: A limit might not exist for some functions at particular values of x. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right. **EXAMPLES OF LIMITS THAT DO NOT EXIST:** $\lim_{x\to 0} \frac{1}{x^2} = \infty$ $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$ *does not exist* $\lim_{x\to 0} \frac{\lvert x\rvert}{x}$ *does not exist* $\lim_{x\to 0} \frac{1}{x}$ *does not exist*
	LO 1.1B: Estimate limits of functions.	**EK 1.1B1:** Numerical and graphical information can be used to estimate limits.

Note: In the Concept Outline, subject matter that is included only in the BC course is indicated with blue shading.

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Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 1.1: The concept of a limit can be used to understand the behavior of functions. *(continued)*	**LO 1.1C:** Determine limits of functions.	**EK 1.1C1:** Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules.
		EK 1.1C2: The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.
		EK 1.1C3: Limits of the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$ may be evaluated using L'Hospital's Rule.
	LO 1.1D: Deduce and interpret behavior of functions using limits.	**EK 1.1D1:** Asymptotic and unbounded behavior of functions can be explained and described using limits.
		EK 1.1D2: Relative magnitudes of functions and their rates of change can be compared using limits.
EU 1.2: Continuity is a key property of functions that is defined using limits.	**LO 1.2A:** Analyze functions for intervals of continuity or points of discontinuity.	**EK 1.2A1:** A function f is continuous at $x = c$ provided that $f(c)$ exists, $\lim_{x \to c} f(x)$ exists, and $\lim_{x \to c} f(x) = f(c)$.
		EK 1.2A2: Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains.
		EK 1.2A3: Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.
	LO 1.2B: Determine the applicability of important calculus theorems using continuity.	**EK 1.2B1:** Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.

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Big Idea 2: Derivatives

Using derivatives to describe the rate of change of one variable with respect to another variable allows students to understand change in a variety of contexts. In AP Calculus, students build the derivative using the concept of limits and use the derivative primarily to compute the instantaneous rate of change of a function. Applications of the derivative include finding the slope of a tangent line to a graph at a point, analyzing the graph of a function (for example, determining whether a function is increasing or decreasing and finding concavity and extreme values), and solving problems involving rectilinear motion. Students should be able to use different definitions of the derivative, estimate derivatives from tables and graphs, and apply various derivative rules and properties. In addition, students should be able to solve separable differential equations, understand and be able to apply the Mean Value Theorem, and be familiar with a variety of real-world applications, including related rates, optimization, and growth and decay models.

Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 2.1: The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies.	**LO 2.1A:** Identify the derivative of a function as the limit of a difference quotient.	**EK 2.1A1:** The difference quotients $\frac{f(a+h)-f(a)}{h}$ and $\frac{f(x)-f(a)}{x-a}$ express the average rate of change of a function over an interval.
		EK 2.1A2: The instantaneous rate of change of a function at a point can be expressed by $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ or $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$, provided that the limit exists. These are common forms of the definition of the derivative and are denoted $f'(a)$.
		EK 2.1A3: The derivative of f is the function whose value at x is $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ provided this limit exists.
		EK 2.1A4: For $y=f(x)$, notations for the derivative include $\frac{dy}{dx}$, $f'(x)$, and y'.
		EK 2.1A5: The derivative can be represented graphically, numerically, analytically, and verbally.
	LO 2.1B: Estimate derivatives.	**EK 2.1B1:** The derivative at a point can be estimated from information given in tables or graphs.

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Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 2.1: The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies. *(continued)*	**LO 2.1C:** Calculate derivatives.	**EK 2.1C1:** Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
		EK 2.1C2: Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
		EK 2.1C3: Sums, differences, products, and quotients of functions can be differentiated using derivative rules.
		EK 2.1C4: The chain rule provides a way to differentiate composite functions.
		EK 2.1C5: The chain rule is the basis for implicit differentiation.
		EK 2.1C6: The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.
		EK 2.1C7: (BC) Methods for calculating derivatives of real-valued functions can be extended to vector-valued functions, parametric functions, and functions in polar coordinates.
	LO 2.1D: Determine higher order derivatives.	**EK 2.1D1:** Differentiating f' produces the second derivative f'', provided the derivative of f' exists; repeating this process produces higher order derivatives of f.
		EK 2.1D2: Higher order derivatives are represented with a variety of notations. For $y = f(x)$, notations for the second derivative include $\frac{d^2y}{dx^2}$, $f''(x)$, and y''. Higher order derivatives can be denoted $\frac{d^ny}{dx^n}$ or $f^{(n)}(x)$.

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Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 2.2: A function's derivative, which is itself a function, can be used to understand the behavior of the function.	**LO 2.2A:** Use derivatives to analyze properties of a function.	**EK 2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.
		EK 2.2A2: Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.
		EK 2.2A3: Key features of the graphs of f, f', and f'' are related to one another.
		EK 2.2A4: (BC) For a curve given by a polar equation $r = f(\theta)$, derivatives of r, x, and y with respect to θ and first and second derivatives of y with respect to x can provide information about the curve.
	LO 2.2B: Recognize the connection between differentiability and continuity.	**EK 2.2B1:** A continuous function may fail to be differentiable at a point in its domain.
		EK 2.2B2: If a function is differentiable at a point, then it is continuous at that point.
EU 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change.	**LO 2.3A:** Interpret the meaning of a derivative within a problem.	**EK 2.3A1:** The unit for $f'(x)$ is the unit for f divided by the unit for x.
		EK 2.3A2: The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable.
	LO 2.3B: Solve problems involving the slope of a tangent line.	**EK 2.3B1:** The derivative at a point is the slope of the line tangent to a graph at that point on the graph.
		EK 2.3B2: The tangent line is the graph of a locally linear approximation of the function near the point of tangency.
	LO 2.3C: Solve problems involving related rates, optimization, rectilinear motion, **(BC)** and planar motion.	**EK 2.3C1:** The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.
		EK 2.3C2: The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.
		EK 2.3C3: The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.
		EK 2.3C4: (BC) Derivatives can be used to determine velocity, speed, and acceleration for a particle moving along curves given by parametric or vector-valued functions.

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Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change. *(continued)*	**LO 2.3D:** Solve problems involving rates of change in applied contexts.	**EK 2.3D1:** The derivative can be used to express information about rates of change in applied contexts.
	LO 2.3E: Verify solutions to differential equations.	**EK 2.3E1:** Solutions to differential equations are functions or families of functions.
		EK 2.3E2: Derivatives can be used to verify that a function is a solution to a given differential equation.
	LO 2.3F: Estimate solutions to differential equations.	**EK 2.3F1:** Slope fields provide visual clues to the behavior of solutions to first order differential equations.
		EK 2.3F2: (BC) For differential equations, Euler's method provides a procedure for approximating a solution or a point on a solution curve.
EU 2.4: The Mean Value Theorem connects the behavior of a differentiable function over an interval to the behavior of the derivative of that function at a particular point in the interval.	**LO 2.4A:** Apply the Mean Value Theorem to describe the behavior of a function over an interval.	**EK 2.4A1:** If a function f is continuous over the interval $[a, b]$ and differentiable over the interval (a,b), the Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.

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Big Idea 3: Integrals and the Fundamental Theorem of Calculus

Integrals are used in a wide variety of practical and theoretical applications. AP Calculus students should understand the definition of a definite integral involving a Riemann sum, be able to approximate a definite integral using different methods, and be able to compute definite integrals using geometry. They should be familiar with basic techniques of integration and properties of integrals. The interpretation of a definite integral is an important skill, and students should be familiar with area, volume, and motion applications, as well as with the use of the definite integral as an accumulation function. It is critical that students grasp the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus — a central idea in AP Calculus. Students should be able to work with and analyze functions defined by an integral.

Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 3.1: Antidifferentiation is the inverse process of differentiation.	**LO 3.1A:** Recognize antiderivatives of basic functions.	**EK 3.1A1:** An antiderivative of a function f is a function g whose derivative is f.
		EK 3.1A2: Differentiation rules provide the foundation for finding antiderivatives.
EU 3.2: The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies.	**LO 3.2A(a):** Interpret the definite integral as the limit of a Riemann sum. **LO 3.2A(b):** Express the limit of a Riemann sum in integral notation.	**EK 3.2A1:** A Riemann sum, which requires a partition of an interval I, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition.
		EK 3.2A2: The definite integral of a continuous function f over the interval $[a, b]$, denoted by $\int_a^b f(x)dx$, is the limit of Riemann sums as the widths of the subintervals approach 0. That is, $\int_a^b f(x)dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f\left(x_i^*\right)\Delta x_i$ where x_i^* is a value in the ith subinterval, Δx_i is the width of the ith subinterval, n is the number of subintervals, and $\max \Delta x_i$ is the width of the largest subinterval. Another form of the definition is $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(x_i^*\right)\Delta x_i$, where $\Delta x_i = \frac{b-a}{n}$ and x_i^* is a value in the ith subinterval.
		EK 3.2A3: The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.

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Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 3.2: The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies. *(continued)*	**LO 3.2B**: Approximate a definite integral.	**EK 3.2B1**: Definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, and verbally.
		EK 3.2B2: Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.
	LO 3.2C: Calculate a definite integral using areas and properties of definite integrals.	**EK 3.2C1**: In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.
		EK 3.2C2: Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.
		EK 3.2C3: The definition of the definite integral may be extended to functions with removable or jump discontinuities.
	LO 3.2D: (BC) Evaluate an improper integral or show that an improper integral diverges.	**EK 3.2D1: (BC)** An improper integral is an integral that has one or both limits infinite or has an integrand that is unbounded in the interval of integration.
		EK 3.2D2: (BC) Improper integrals can be determined using limits of definite integrals.
EU 3.3: The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration.	**LO 3.3A**: Analyze functions defined by an integral.	**EK 3.3A1**: The definite integral can be used to define new functions; for example, $f(x)=\int_0^x e^{-t^2}\,dt$.
		EK 3.3A2: If f is a continuous function on the interval $[a, b]$, then $\frac{d}{dx}\left(\int_a^x f(t)dt\right)=f(x)$, where x is between a and b.
		EK 3.3A3: Graphical, numerical, analytical, and verbal representations of a function f provide information about the function g defined as $g(x)=\int_a^x f(t)dt$.

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Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 3.3: The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration. *(continued)*	**LO 3.3B(a):** Calculate antiderivatives.	**EK 3.3B1:** The function defined by $F(x)=\int_a^x f(t)dt$ is an antiderivative of f.
	LO 3.3B(b): Evaluate definite integrals.	**EK 3.3B2:** If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then $\int_a^b f(x)dx = F(b)-F(a)$.
		EK 3.3B3: The notation $\int f(x)dx = F(x)+C$ means that $F'(x)=f(x)$, and $\int f(x)dx$ is called an indefinite integral of the function f.
		EK 3.3B4: Many functions do not have closed form antiderivatives.
		EK 3.3B5: Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables, **(BC)** integration by parts, and nonrepeating linear partial fractions.
EU 3.4: The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation.	**LO 3.4A**: Interpret the meaning of a definite integral within a problem.	**EK 3.4A1:** A function defined as an integral represents an accumulation of a rate of change.
		EK 3.4A2: The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.
		EK 3.4A3: The limit of an approximating Riemann sum can be interpreted as a definite integral.
	LO 3.4B: Apply definite integrals to problems involving the average value of a function.	**EK 3.4B1:** The average value of a function f over an interval $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)dx$.
	LO 3.4C: Apply definite integrals to problems involving motion.	**EK 3.4C1:** For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle’s displacement over the interval of time, and the definite integral of speed represents the particle’s total distance traveled over the interval of time.
		EK 3.4C2: (BC) The definite integral can be used to determine displacement, distance, and position of a particle moving along a curve given by parametric or vector-valued functions.

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Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 3.4: The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation. *(continued)*	**LO 3.4D:** Apply definite integrals to problems involving area, volume, **(BC)** and length of a curve.	**EK 3.4D1:** Areas of certain regions in the plane can be calculated with definite integrals. **(BC)** Areas bounded by polar curves can be calculated with definite integrals.
		EK 3.4D2: Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals.
		EK 3.4D3: (BC) The length of a planar curve defined by a function or by a parametrically defined curve can be calculated using a definite integral.
	LO 3.4E: Use the definite integral to solve problems in various contexts.	**EK 3.4E1:** The definite integral can be used to express information about accumulation and net change in many applied contexts.
EU 3.5: Antidifferentiation is an underlying concept involved in solving separable differential equations. Solving separable differential equations involves determining a function or relation given its rate of change.	**LO 3.5A:** Analyze differential equations to obtain general and specific solutions.	**EK 3.5A1:** Antidifferentiation can be used to find specific solutions to differential equations with given initial conditions, including applications to motion along a line, exponential growth and decay, **(BC)** and logistic growth.
		EK 3.5A2: Some differential equations can be solved by separation of variables.
		EK 3.5A3: Solutions to differential equations may be subject to domain restrictions.
		EK 3.5A4: The function F defined by $F(x)=c+\int_a^x f(t)dt$ is a general solution to the differential equation $\frac{dy}{dx}=f(x)$, and $F(x)=y_0+\int_a^x f(t)dt$ is a particular solution to the differential equation $\frac{dy}{dx}=f(x)$ satisfying $F(a)=y_0$.
	LO 3.5B: Interpret, create, and solve differential equations from problems in context.	**EK 3.5B1:** The model for exponential growth and decay that arises from the statement "The rate of change of a quantity is proportional to the size of the quantity" is $\frac{dy}{dt}=ky$.
		EK 3.5B2: (BC) The model for logistic growth that arises from the statement "The rate of change of a quantity is jointly proportional to the size of the quantity and the difference between the quantity and the carrying capacity" is $\frac{dy}{dt}=ky(a-y)$.

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